Midterm Examination I

Please Place your name of the BACK of the LAST PAGE of the examination!

There are three questions on this examination. The first question is a series of 6 (true-false) questions, each worth 5 points each for a total of 30 points. All other questions are open-ended with point values as indicated. Please answer all questions and show all work on this examination paper. You may use the back of the sheets if necessary.

**Question I: 30 points:**

T  F  a) For an absorbing Markov Chain, the fundamental matrix is given as $N = [I - Q]^{-1}$ and the components $n_{ij}$ of $N$ represent the normalized probability that the DTMC is in the transient state $j$ given that it started in the transient state $i$.

T  F  b) A transient state in a DTMC will be visited only a finite number of times.

T  F  c) For the expected first passage time, the following formula $\mu_{ij} = 1 + \sum_{k \neq j} p_{ik} \mu_{kj}$ indicates that the expected first passage time is at least 1 time step.

T  F  d) The following steady-state equations for a Continuous Time Markov Chain (CTMC) are called the “global balance” equations $\pi_j q_j = \sum_{i \neq j} \pi_i q_{ij} \forall j$

T  F  e) If the number of identical parts in a plant is 300, the chance of each part failing is 1%. The gain by replacement is $10,000 while the loss by not using it is$2000. The optimal number of spare parts to be kept in stock is about 5?

T  F  f) Two random variables $\{X, Y\}$ are dependent if their joint density function is not the product of their marginal density functions.
Question II: 35 points
Janie owns Jainie's Java Shoppe and likes to converse with her customers. She has a keen appreciation for precise and elegant English. One day, a Mutant Creepazoid wanders in for a cup of coffee. The creepazoid's brain has all been destroyed by too much television and other toxic substances and this has resulted in his limited conversation ability. In fact, the conversation of the Creepazoid consists primarily of the following phrases:

1) Ya know.
2) It was weird.
3) Like, I dunno man.
4) Bummer.
5) It was awesome.
6) F-word.

The appearance of these phrases in the mutant creepazoid's speech follows a DTMC (if ever there was a case for lack of long term memory, this is it!) with the following transition matrix:

\[
P = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & .6 & .1 & .1 & .1 & .1 & 0 \\
2 & .3 & 0 & .3 & .2 & .1 & .1 \\
3 & .3 & .5 & 0 & .1 & .1 & 0 \\
4 & .1 & .2 & .2 & 0 & .1 & .4 \\
5 & .2 & .2 & 0 & .1 & 0 & .5 \\
6 & .2 & .2 & .2 & .2 & 0 & 1
\end{pmatrix}
\]

a) Is this an irreducible, ergodic DTMC? Please conclusively show this.

b) If the first phrase out of his mouth is equally likely to be any of the phrases other than the F-word, then what is the probability that the next phrase out of his mouth is the F-word.

c) If you want to know the long term frequency that each phrase appears in the creepazoid's conversation, what set of equations should you set up. Explicitly set up all the equations. Please do not attempt to solve the equations!
Section 1 (continued)
Question II (continued)
Question III: 35 points
Consider the following transition matrix, which represents a DTMC with state space \{a, b, c, d, e\}

\[
P = \begin{pmatrix}
\frac{3}{10} & 0 & 0 & \frac{7}{10} & 0 \\
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & \frac{1}{5} & \frac{2}{5} & 0 & \frac{2}{5}
\end{pmatrix}
\]

i) Draw the state transition diagram.

ii) List the recurrent states.

iii) List the irreducible classes.

iv) List the transient states.

v) Calculate the limiting probabilities \(\lim_{n \to \infty} P^{(n)}\) for the various states.

vi) What are the periods for the various states.
Question III (continued)
Question III (continued)
Question III (continued)