Conditional Probability and Conditioning

#1 (10 pts.) An urn contains three white, six red and five black balls. Six of these balls are randomly selected from the urn. Let $X$ and $Y$ denote respectively the number of white and black balls selected. Compute the conditional probability mass function (pmf) of $X$ given that $Y = 3$. Also compute $E[X|Y = 1]$.

#2 (10 pts.) Consider the trapped miner discussed in lecture #4. Let $N$ denote the total number of doors selected before the miner reaches safety. Also, let $T_i$ denote the travel time corresponding to the $i$th choice, $i \geq 1$. Again let $X$ denote the time when the miner reaches safety.
   
   a) Given an identity that relates $X$ to $N$ and the $T_i$.
   b) What is $E[N]$?
   c) What is $E[T_N]$?

#3 (10 pts.) Data indicate that the number of traffic accidents in Amherst on a rainy day is a Poisson RV with mean 9, whereas on a dry day it is a Poisson RV with mean 3. Let $X$ denote the number of traffic accidents tomorrow. If it will rain tomorrow with probability .6, find:
   
   a) $E[X]$;
   b) $P\{X = 0\}$;
   c) $\text{Var}(X)$.

DTMC Formulation

#4 (10 pts.) Suppose that whether or not it rains today depends upon previous weather conditions through the last three days.
   
   a) Show that this system may be analyzed by using a DTMC.
   b) How many states are needed?
   c) Suppose that if it has rained for the past three days, then it will rain today with probability 0.8; if it did not rain for any of the past three days, then it will rain with probability 0.2; and in any other case, the weather today will, with probability 0.6, be the same as the weather yesterday. Determine $P$ for this DTMC.

#5 (10 pts.) Problem #2 on page 54 of our textbook

#6 (10 pts.) Problem #1 on page 58 of our textbook

#7 (10 pts.) Problem #7 on page 59 of our textbook