Homework #4: Due in two weeks

1. (10 pts.) Problem #1 (page 180)

2. (10 pts.) Problem #3 (page 180)

3. (10 pts.) Problem #7 (page 181)

4. (10 pts.) Consider the Discrete Time MDP with state space \{1, 2, 3\} and action space \{1, 2, 3\} with the following probability transition matrices:

\[
P(1) = \begin{pmatrix} 1 & 0 & 0 \\ .3 & .7 & 0 \\ .1 & .8 & .1 \end{pmatrix} \quad P(2) = \begin{pmatrix} .5 & .5 & 0 \\ 0 & .5 & .5 \\ 0 & 0 & .5 \end{pmatrix} \quad P(3) = \begin{pmatrix} .5 & .5 & 0 \\ .8 & 0 & .2 \\ 0 & .3 & .7 \end{pmatrix}
\]

and the following cost matrix:

\[
R = \begin{pmatrix} 30 & 10 & 40 \\ 30 & 10 & 5 \\ 20 & 50 & 10 \end{pmatrix}
\]

i) Set up the Linear Program formulation for the optimal MDP policy.

ii) Compute the optimal policy with the simplex method. You may use a computer to solve the LP but please include the print-out of the setup and solution.

5. (10 pts.) Consider a network with three arcs as shown in the following figure, Figure 1:

![Network Diagram](image)

Figure 1: 3-node Network

Let \(X_i\) represent the length of arc \(i\). Suppose that \(X_i \sim \text{exp}(\lambda_i)\) are independent RVs. Compute the distribution of the shortest path from A to C.

6. (10 pts.) Please do problem #13 in your text, p. 233.

7. (10 pts) A two-dimensional Poisson process is a process of randomly occurring points in a Euclidean plane such that

i) the number of points in a region of area \(A\) is \(P(\lambda A)\) and

ii) the number of points in disjoint regions are independent of each other.

Consider an arbitrary point “a” in this plane. Let \(X_1\) denote the distance from a to the nearest point. Compute the density of \(X_1\). Hint: compute \(P(X_1 > t)\) first.