Homework #3

1. Solve the following problem:

Minimize $x_1 - 8x_2$

\[
\begin{align*}
  x_1 + x_2 & \geq 1 \\
  -x_1 + 6x_2 & \leq 3 \\
  x_2 & \leq 2 \\
  x_1, x_2 & \geq 0
\end{align*}
\]

a) Graphically.

b) By the Two-Phase Simplex Method. Show that the points generated by Phase I correspond to basic solutions of the original system.

2. Show how Phase I of the Simplex method can be used to solve $n$ simultaneous linear equations in $n$ unknowns. Show how the following cases can be detected:

a) Inconsistency of the system.

b) Redundancy of the equations.

c) Unique solution.

Also show how the inverse matrix corresponding to the system of equations can be found in part (c). Illustrate using the following system:

\[
\begin{align*}
  x_1 + 2x_2 + x_3 & = 4 \\
  -x_1 - x_2 + 2x_3 & = 3 \\
  3x_1 + 5x_2 & = 5
\end{align*}
\]

3. Solve the following problem by the Big-M method:

Minimize $-2x_1 - 4x_2 - 4x_3 + 3x_4$

\[
\begin{align*}
  2x_1 + x_2 + x_3 & = 4 \\
  x_1 + 4x_2 + 4x_4 & = 8 \\
  x_1, x_2, x_3, x_4 & \geq 0
\end{align*}
\]
4. Solve the following problem by the lexicographic simplex method, then repeat solving the problem with Bland’s rule:

Minimize $-x_1 - 2x_2 - x_3$

$x_1 + 4x_2 + 6x_3 \leq 4$
$-x_1 + x_2 + 4x_3 \leq 1$
$x_1 + 3x_2 + x_3 \leq 6$
$x_1, x_2, x_3 \geq 0$

5. Solve the following problem by the Revised Simplex Algorithm:

Minimize $x_1 + 6x_2 - 7x_3 + x_4 + 5x_5$

$x_1 - \frac{3}{4}x_2 + 2x_3 - \frac{1}{4}x_4 = 5$
$\frac{1}{4}x_2 + 3x_3 - \frac{3}{4}x_4 + x_5 = 5$
$x_1, x_2, x_3, x_4, x_5 \geq 0$

6. Solve the previous problem by the Product Form of the Inverse.

7. Solve the previous problem in Exercise #5 by the LU-Decomposition approach.