1. Consider the following optimal tableau of a maximization problem where the constraints are of the \( \leq \) type:

\[
\begin{pmatrix}
    z & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & RHS \\
    0 & 0 & 0 & 2 & 0 & 2 & \frac{1}{10} & 2 & \theta \\
    1 & 1 & 0 & 0 & -1 & 0 & \frac{1}{2} & \frac{1}{5} & -1 & 2 \\
    0 & 1 & 0 & 2 & 1 & -1 & 0 & \frac{1}{2} & 3 \\
    0 & 0 & 1 & -1 & -2 & 5 & -\frac{3}{10} & 2 & 1 \\
\end{pmatrix}
\]

a) Find the optimal objective function value \( \theta \)

b) Would the solution be altered if a new activity \( x_9 \) with coefficients \((2, 0, 3)^t\) in the constraints, and price of 5 were added to the problem?

How large can \( b_1 \) (the first constraint resource) be made without violating feasibility?

2. Consider the following problem below where the cost vector is modified in the direction \((-1, 1)\). Using parametric analysis on the cost vector, find the sequence of optimal solutions:

\[
\text{Minimize } x_1 - 3x_2 \\
2x_1 + 3x_2 \leq 6 \\
x_1 - 2x_2 \geq -2 \\
x_1, x_2 \geq 0
\]

3. You are given the following linear programming problem:

\[
\text{Minimize } Z = 3x_1 - 4x_2 + 3x_3 + 5x_4
\]

subject to:

\[
\begin{align*}
-x_1 + 4x_2 - 2x_3 - x_4 & \geq 0 \\
-x_1 - x_2 - x_3 - x_4 & \geq -1 \\
2x_1 - 3x_2 + 3x_3 + 2x_4 & \geq 1 \\
x_1, x_2, x_3, x_4 & \geq 0
\end{align*}
\]
Please answer the following questions:

a) Utilizing the primal and dual properties of the problem, please place this problem into a format suitable for Khachian’s Ellipsoid Algorithm.

b) You are given the following set of inequalities:

\[-2x_1 - 3x_2 \leq -3\]
\[x_1 - 2x_2 \leq -1\]
\[x_1 + 3x_2 \leq 5\]
\[-5x_1 + 3x_2 \leq 1\]

Using a starting sphere of radius $\sqrt{5}$, centered at the origin, execute 2 – 3 iterations of Khachian’s algorithm. Show all details of the calculations. You may use computer routines such as MATLAB, Mathematica, or MAPLE to help do your calculations.

c) Please illustrate the progress of Khachian’s algorithm on the set of inequalities in part b.

4. You are given the following linear programming problem:

Minimize $Z = -2x_1 - 2x_2 - 4x_3$

subject to:

\[x_1 - x_2 + x_3 + x_4 = 1\]
\[2x_1 + 2x_2 + 4x_3 + x_5 = 5\]
\[x_1, x_2, x_3, x_4, x_5 \geq 0\]

Please answer the following questions:

a) Using a starting solution of $\bar{x}^0 = (0.5, 0.5, 0.5, 0.5, 1.0)^t$ to start the process, employ Dikin’s algorithm for two iterations and determine the resulting solution and objective function value. Please carry out the steps of Dikin’s algorithm by hand showing all results of the calculations for each step. You may use computer routines such as MATLAB, Mathematica, or MAPLE to help do your calculations.

b) Compare your solution with one from the Simplex algorithm (you can use a computer program of the Simplex for this if you wish). Can you explain the difference between the solution you get from Dikin’s algorithm and the one for the simplex algorithm.