Midterm Examination II

Question #1: ___________ /25
Question #2: ___________ /15
Question #3: ___________ /25
Question #4: ___________ /35

Grand Total $\sum$: ___________ /100

Examination Number #: ___________
Midterm Examination II

Please Place your name of the BACK of the LAST PAGE of the examination!

There are four questions on this examination. The first question is a series of 5 (true-false) questions, each worth 5 points each for a total of 25 points. The other three questions are open-ended with point values as indicated. Please answer all questions and show all work on this examination paper. Please answer all questions and show all work on your examination paper. You may use the back of the sheets if necessary

Question I 25 points:

In the following true-false questions, answer all questions, there is no penalty for guessing. Please circle the appropriate response!

T F a) In Dynamic Programming, it is usually more difficult to define the stages rather than the states.

T F b) If customers arrive at an M/M/1 system with an expected inter-arrival time of 25 minutes and service times have a mean of 30 minutes, then in this situation, the queue will grow without bound.

T F c) The principle of optimality guarantees that future decisions are made independently of previous made decisions.

T F d) For an M/G/1 queue with λ and μ fixed, the value of Lq with an exponential service time distribution is twice as large as with constant service times.

T F e) In a Jackson network, we must have sjμj < λj ∀j nodes
Question II: 15 points

The time $T$ required to repair a machine is an exponentially distributed random variable with mean of $\frac{1}{2}$ (hours).

a) What is the probability that a repair time exceeds $\frac{1}{2}$ hour?

b) What is the probability that a repair takes at least $12\frac{1}{2}$ hours, given that its duration exceeds 12 hours?
Question III: 25 points

Consider a discrete time Markov Decision Process with state space \{1, 2, 3, 4\} and decision space \{1, 2\} with the following transition probability matrices:

\[
P(1) = \begin{pmatrix}
.25 & .75 & 0 & 0 \\
0 & .25 & .75 & 0 \\
0 & 0 & .25 & .75 \\
.75 & 0 & 0 & .25
\end{pmatrix} \quad P(2) = \begin{pmatrix}
.25 & 0 & 0 & .75 \\
.75 & .25 & 0 & 0 \\
0 & .75 & .25 & 0 \\
0 & 0 & .75 & .25
\end{pmatrix}
\]

and the following cost matrix:

\[
\begin{pmatrix}
10 & 10 \\
20 & 30 \\
30 & 20 \\
40 & 10
\end{pmatrix}
\]

Formulate a linear programming model of this DTMEDP:

i) Please specify the decision variables.

ii) Specify the objective function.

iii) Specify the constraints.
Question IV: 35 points

A downtown Boston car service station has facilities for a maximum of 4 cars being serviced or waiting for service on its premises. Past experience indicates that no potential customers join the queue once these 4 places are filled. The arrival rate of customers is 24 per hour during off-peak hours, and the input process is approximately Poisson. The service times are exponential with a mean of 3 minutes.

a) Draw the and label the rate diagram for this process.

b) Find the steady state probabilities for this system.

c) What is the average idle time of the attendant?

d) What is the fraction of customers lost? If the average profit per customer is $0.80, what is the lost profit per hour?

e) What is the average waiting time of an arrival?