The first 5 problems are from Chapter 24 of our textbook.

#1 (10 pts.) Problem #24-1:

#2 (10 pts.) Problem #24-2(a-d):

#3 (10 pts.) Problem #24-3:

#4 (10 pts.) Problem #24-6:

#5 (10 pts.) Problem #24-10:

#6 (10 pts.) Suppose that a RV $X$ has density $f$ given by:

$$f(x) = \begin{cases} 
\frac{2}{a^2}(a-x) & \text{for } 0 \leq x \leq a \\
0 & \text{for } x \geq a 
\end{cases}$$

where $a$ is a positive constant.

(a) Show that the cumulative distribution function $F$ is given by:

$$F(x) = \begin{cases} 
\frac{x}{a}(2 - \frac{x}{a}) & \text{for } 0 \leq x \leq a \\
0 & \text{for } x \geq a 
\end{cases}$$

(b) Show that the mean life time $\mu$ is

$$\mu = \frac{a}{3}$$

and its variance is

$$\sigma^2 = \frac{a^2}{18}$$

(c) Sketch the graphs of $f(x), F(x)$ indicating slopes, etc.

(d) Take $a = 2$ hours and compute the pr. that the life time is between 30 and 60 minutes.

#7 (10 pts.) Assume that the RV of a battery in a flashlight has the uniform distribution with a constant density $1/L$.

(a) Suppose the mean time was found to be 25 days. Determine $L$.

(b) Suppose the battery has already lasted 30 days. Find the probability that the flashlight will continue to operate (with the same battery) for more than one week.