Midterm Examination I

Question #1: 

Question #2: 

Question #3: 

Question #4: 

Grand Total $\sum$: 

Examination Number #: 16
Midterm Examination

Please Place your name of the BACK of the LAST PAGE of the examination!

There are four questions on this examination. The first question is a series of 5 (true-false) questions, each worth 4 points each for a total of 20 points. The other three questions are open-ended with point values as indicated. Please answer all questions and show all work on this examination paper. Please answer all questions and show all work on your examination paper. You may use the back of the sheets if necessary.

Question I 20 points:

In the following true-false questions, answer all questions, there is no penalty for guessing. Please circle the appropriate response!

T  F  a) If the density of $X$ equals $Ce^{-2x}$, $0 \leq x < \infty$ and is $0$, $x < 0$, then we know that $C = 1/2$.

T  F  b) Let $X$ and $Y$ be two independent random variables with densities $f_1$ and $f_2$ respectively. The sum of the two random variables is then given by $Z = X + Y$.

T  F  c) Once a system enters a transient state in a Markov chain it remains there indefinitely.

T  F  d) Suppose one die is thrown many times. Let $X_n$ = the number showing on the $n$th toss. $\{X_n : n = 1, 2, \ldots\}$ represents a Markov Chain.

T  F  e) If the number of identical parts in a plant is 300, the chance of each part failing is 1%. The gain by replacement is $7,000 while the loss by not using it is $3000. The optimal number of spare parts to be kept in stock is about 5?
Question II: 25 points

A machine shop operates two identical machines, which are supervised by one operator. Each machine requires the operator’s attention at random points in time. The probability that the machine requires service in a period of 5 minutes is $p = 0.4$. The operator is able to service the machine in 5 minutes. Let us approximate the situation by assuming that a machine requires service always at the beginning of a 5 minute period.

a) Construct a transition diagram for this problem and find the transition matrix associated with it.

b) If both machines are operating at 8AM, find the marginal probabilities after 5 minutes and 10 minutes.

c) Find the steady state probabilities for this process.
Question III: 25 points

A video recorder manufacturer is so certain of its quality control, that it is offering a complete replacement warranty if the set fails within two years. Based upon complied data, the company has noted that only 1 percent of its recorders fail during the first year, whereas 5 percent of the recorders that survive the first year will fail during the second year. The warranty does not cover replaced recorders.

a) Formulate the the evolution of the status of a recorder as a DTMC whose states include two absorption states that involve needing to honor the warranty or having the recorder survive the warranty period. Then construct the transition matrix.

b) Find the probability that the manufacturer will have to honor the warranty.
Question IV: 30 points

Assume that the probability of rain tomorrow is $\alpha$ if it is raining today, and assume that the probability of its being clear tomorrow is $\beta$, if it is clear today.

(a) Determine the one-step transition matrix of the DTMC.

(b) Find the two-step transition matrix.

(c) Find the steady-state probabilities.